

斜齿轮齿面柔度矩阵与修形的有限元计算

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【摘要】 提出了一套斜齿轮修形用的有限元计算程序。该程序自动生成轮齿有限元网格和齿面接触线,计算得到接触线上结点的柔度矩阵;然后考虑支承结构与齿轮的综合变形,由数学规划获得最佳齿面修形。计算与实验结果对比证明该程序操作简便、计算精度高,具有工程应用价值。

主题词: 斜齿圆柱齿轮 柔度 修形 有限元方法

分类号: V 261.27 TH 132.4

1 轮齿有限元网格自动生成与齿面结点柔度矩阵

在工业发达国家的航空齿轮设计中,齿轮强度与修形的有限元分析已是常规手段,但其繁琐的数据处理使不少工程技术人员望而却步。为此,有限元网格自动生成技术得到了普遍的重视。由于齿轮网格的划分必须与轮齿接触相适应,现已发表的网格自动生成程序^[1]往往是先生成齿面接触线,再根据接触线划分网格,划分需要重复进行,且操作人员也需要具有一定的啮合理论知识;划分时须迁就接触线,使单元畸变,影响了计算精度。

先根据齿轮端面参数,按直齿轮展成齿廓^[2],在距端面 L 处,将齿廓平行扭转 $\theta(L)$ 角,得到斜齿轮齿面。 $\theta(L)$ 按下式计算:

$$\theta(L) = L \cdot \tan \beta_b / r_b$$

式中 β_b 为基圆螺旋角, r_b 为基圆半径。在平行于端面的各截面中,齿廓坐标可由相应的端面齿廓坐标旋转而成。结构参数按回转体方式生成。主、被动齿轮的网格分别生成,并与啮合无关,故划分容易,且可尽量划分均匀,以提高有限元计算精度。

按上述方法生成了一对齿轮的网格,由计算机绘出示于图1中,单元类型为20结点等参元,

齿轮参数为:齿数 $Z_1=24$, $Z_2=48$;模数 $m_n=4\text{mm}$;压力角 $\alpha_0=20^\circ$;螺旋角 $\beta=16.26^\circ$;齿宽 $B=20\text{mm}$;旋向小轮右旋。齿面结点柔度矩阵 $[F_p]_k$ ($k=1,2$,分别表示主、被动齿轮)中任一元素 f_{pij} 定义为:在齿面 i 结点施加一单位法向力时,在齿面 j 结点产生的法向位移。在齿面各结点依次施加单位法向载荷,即可计算得到 $[F_p]_k$ 。

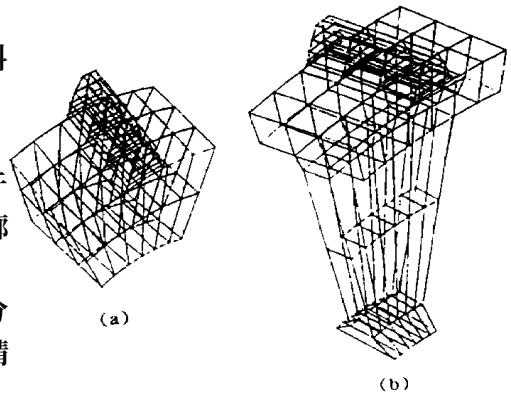


图1 有限元网格生成((a)小齿轮(b)大齿轮)

2 接触线结点柔度矩阵及斜齿轮修形计算

首先须在被计算齿轮副的啮合面上根据啮合重合度生成接触线。生成过程从啮入端起始,此时接触线为一个点(见图 2 中啮合面的右上角),再每相距一个法节 p_n 逐条生成同时啮合的各条接触线,在本例中共有 3 条;然后根据需要计算的齿轮转角位置等分 p_n ,生成逐次啮合的各条接触线,再等分各条接触线,即可获得接触线上的结点,每个结点为两齿面在啮合过程中的接触点,此过程由计算机实现。

接触线结点柔度矩阵 $[F]_k$ 由原网格结点柔度矩阵 $[F_p]_k$ 在啮合面上经二元三点插值获得。

插值在两个齿轮上分别进行,对于本文算例,两个齿轮的插值节点与插值点位置如图 2 所示。图中 $\cdot$ 为网格结点(节点), $\times$ 为接触线结点(插值点),斜线为齿面接触线。考虑到受载齿上的载荷对邻近齿的变形影响极小,因此, $[F]_k$ 处理成对角块矩阵,即:

$$[F]_k = \begin{bmatrix} [F_1] & & 0 \\ & [F_2] & \\ 0 & & \ddots \\ & & & [F_l] \end{bmatrix}$$

式中 l 为接触线总条数,每个子块表示一条接触线,它的阶数为接触线上的结点数,任意两条接触线之间无联系。齿轮副接触结点柔度矩阵由两个齿轮的柔度矩阵叠加而成:

$$[F] = [F]_1 + [F]_2$$

齿轮修形主要是为了克服振动、冲击及由于支承变形而引起的偏载,因此柔度矩阵中还须包括支承变形,作者已详细推导了叠加方法^[3],本文被计算齿轮副的支承情况如图 3 所示。

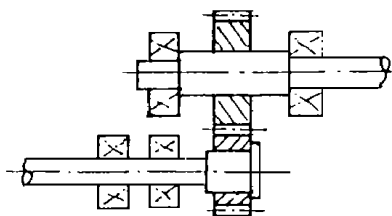


图 3 齿轮副的支承状况

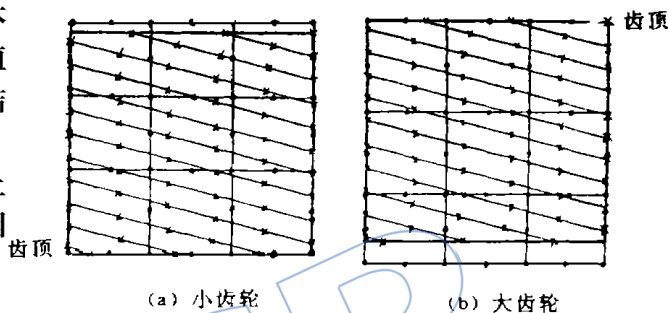


图 2 接触线结点柔度在啮合面上的插值

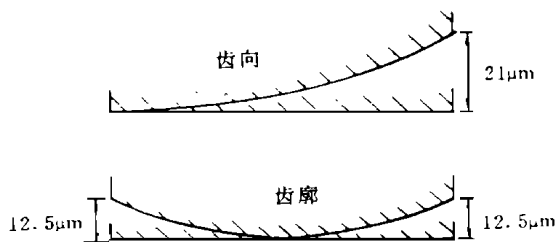


图 4 最佳齿面修形

假定修形量曲面可用二次曲面表示为: $z = a_1x^2 + a_2y^2 + b_1x + b_2y + c_0$ 。式中 x 为沿齿宽方向坐标, y 为沿齿廓方向坐标, 均在啮合面上度量; (x, y) 为啮合面上一点, 对应工作齿面的一个接触点, $z(x, y)$ 为该点修形量。若共有 n 个接触线结点, 分属 m 个转角位置, 齿面优化修形可表示为数学规划形式^[4](式中符号意义参见文献[4])。

规划中, 约束条件第 1 式表示变形协调条件, 第 2 式选出最大载荷密度, 第 3~5 式限制传动误差幅值, 第 6 式为力平衡条件, 第 8 式为非线性接触条件, 优化目标为均化齿面载荷分布。作者编制了整套微机软件, 只需输入齿轮参数、支承和工作条件、网格划分要求, 在 486 微机上

运行 40 分钟,即可获得齿轮的接触状况与最佳修形(见图 4)。

$$\left\{ \begin{array}{l} m \ln(p_{max}) \\ - [F]\{p\} + [D]\{\alpha\} + [I]_n\{\omega\} = \{E\} + a_1\{x^2\} + a_2\{y^2\} + b_1\{x\} + b_2\{y\} + c\{e\} \\ [H]\{p\} - p_{max}\{e\}_n \leq \{0\}_n \\ [I]_m\{\alpha\} - \alpha_{max}\{e\}_m \leq \{0\}_m \\ [I]_m\{\alpha\} - \alpha_{in}\{e\}_m \geq \{0\}_m \\ \alpha_{max} - \alpha_{in} \leq C_e \cdot \alpha_n \\ [D]^T\{p\} = P\{e\}_m \\ \{p\}, \{\alpha\}, \{\omega\}, a_1, a_2, c, \alpha_{max}, \alpha_{in}, p_{max} \geq 0 \\ \text{或 } p_j = 0, \text{ 或 } \omega_j = 0 \quad (j = 1, 2, \dots, n) \end{array} \right.$$

3 与实验结果的比较

修形前后齿面的载荷分布和传动误差由柔度矩阵经接触分析获得,示于图 5 中,上图为载荷分布,下图为传动误差 δ_r (θ_r 为转角位置)。轴承刚度计算中考虑有 $5\mu\text{m}$ 的间隙,计算输入扭矩为 150Nm ,修形后在载荷分布及传动误差幅值上都有显著的改善。

用本文算例齿轮和图 3 所示支承条件,在齿轮修形前后分别进行了啮合印痕和传动误差的测试,齿轮精度为 5 级,并按图 4 要求进行了修形,试验载荷与计算时相同,加载运行后,修形前啮合印痕约占齿宽的 $2/3$,处于小齿轮支承端;修形后达到全长接触。传动误差

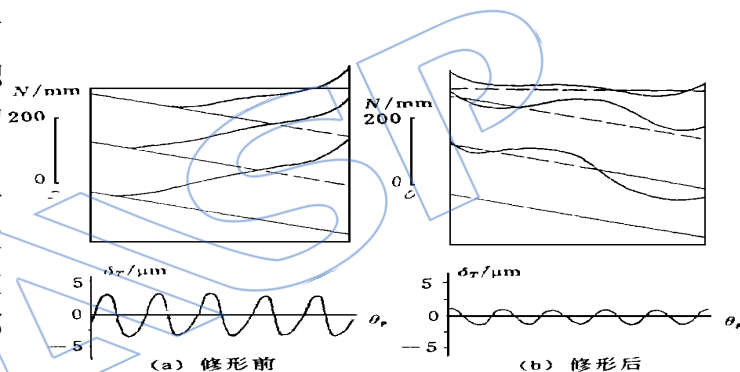


图 5 齿轮修形效果的计算

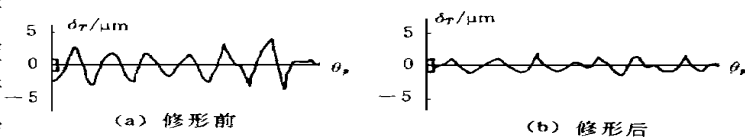


图 6 齿轮修形效果的测量

的测量曲线由计算机绘制,示于图 6 中,修形后其幅值有显著下降,并与计算结果吻合较好。

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TRANSIENT CHARACTERISTICS OF A FLEXIBLE ROTOR SUPPORTED ON NONLINEAR SFD'S THROUGH BISTABLE OPERATION RANGE

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ABSTRACT The transient characteristics of a flexible rotor supported on nonlinear squeeze film dampers (SFD's) during passing through the resonant and bistable operation speed range with constant acceleration are studied theoretically and experimentally. It is shown that the properly designed SFD's have not only excellent steady-state performance, but also better acceleration performance. The maximum vibration amplitude of the rotor system in the acceleration process decreases gradually with increase of rotor angular acceleration, whether for the properly designed SFD's or for the improperly designed SFD's. The stability of multistable operation in the bistable operation speed ranges depends not only on the form of bistable operation, but also on the moving process. It is impossible for the rotor system to pass through the bistable operation speed ranges with smaller amplitudes only by means of accelerating the rotor system.

Key words Rotors Dampers Transient response Dynamic characteristics

CALCULATION ON FLEXIBILITY MATRIX AND MODIFICATION OF HELICAL GEARS BY FEM

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ABSTRACT A computer program is worked out for modification design of helical gears by using Finite Element Method (FEM). At first the finite element mesh of gear tooth is auto-generated without engagement of gears, therefore it can be divided more conveniently and uniformly. The flexibility matrixes of mesh nodes on working flanks of gears are calculated by FEM. Then, the contact lines and contact nodes on the plane of action are generated by computer according to the contact ratio of gear pair. The flexibility matrixes of contact nodes are obtained by interpolation between the mesh nodes on plane of action. Further, the deformation of shafts and bearings is added to the total flexibility matrix which is sum of the flexibility matrixes of both gears. Finally, considering minimization of transmitting error and uniformization of load density as objective functions, an optimal 3-D modification is solved by using mathematical programming technique. Moreover an experiment on modification of a helical gear pair has been done, which showed that the calculated results agreed with experimental data and the carrying behaviour of gear transmission was improved considerably.

Key words Helical gears Flexibility Modification Finite element methods

OPTIMIZATION DESIGN OF WHOLE CONSTRUCTION OF MULTISTAGE DISK

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ABSTRACT The optimization design of a whole construction of multistage disk, which is of great significance in practice, is the further development of optimization design of sin-